Collective LIME: Enhancing the explainability of the explainer

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Thank you to the work of the **EUROYoung** Forum!



Outline

Introduction

• The CLIME methodology

Conclusions

When training a machine learning model, **accuracy** of its predictions matters, as does its **transparency** (Baesens et al., 2003; Panigutti et al., 2023; Rudin et al., 2022)

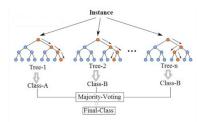


Generally, linear models (e.g., medical scoring systems (Ustun and Rudin, 2016)) are considered to be easy-to-understand, as well as rule-based models (Carrizosa et al., 2021b)



Emilio Carrizosa, Cristina Molero-Río & Dolores Romero Morales

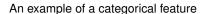
More complex models such as Random Forests (Breiman, 2001) XGBoost (Chen and Guestrin, 2016) and Deep Learning (Goodfellow et al., 2016) are seen as **black boxes**



- Even linear models lose transparency when their complexity increases, as measured, e.g., by # features. Classic methodologies such as LASSO (Tibshirani, 1996) or Best Subset selection (Bertsimas et al., 2016; Hazimeh and Mazumder, 2020) aim at selecting a small subset of features that give a good accuracy
- Similarly, rule-based models lose transparency when the # rules increases (Carrizosa et al., 2025b)

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- ullet Similarly, rule-based models lose transparency when the # rules increases (Carrizosa et al., 2025b)

Sparsity in Generalized Linear Models for categorical data in Carrizosa et al. (2021a)



















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On clustering categories of categorical predictors in generalized linear models

Emilio Carrizosa ^a ⊠ , Marcela Galvis Restrepo ^b 🞗 ⊠ , Dolores Romero Morales ^b ⊠

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Sparsity in Generalized Linear Models for categorical data in Carrizosa et al. (2021a)



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Clustering categories into red and blue clusters





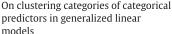












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Sparsity in Generalized Linear Models for categorical data in Carrizosa et al. (2021a)

yields a sparser representation of the variable







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On clustering categories of categorical predictors in generalized linear models

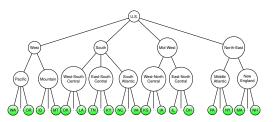
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Sparsity for Linear Models for hierarchical data in Carrizosa et al. (2022b)

An example of a hierarchical categorical feature





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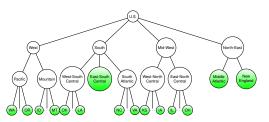


The tree based linear regression model for hierarchical categorical variables

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Sparsity for Linear Models for hierarchical data in Carrizosa et al. (2022b)

Clustering cat. yields a sparser representation





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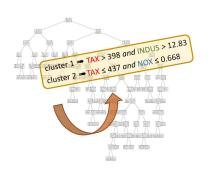


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The tree based linear regression model for hierarchical categorical variables

Enhancing the transparency of rule-based models

If-then rules to explain clusters in Carrizosa et al. (2023)





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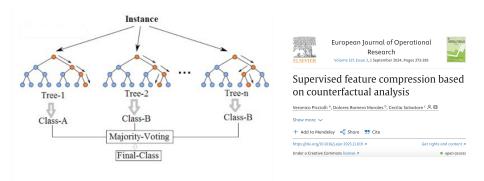


On clustering and interpreting with rules by means of mathematical optimization



Building surrogate interpretable models from opaque ones¹

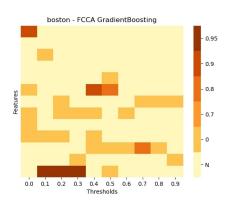
A surrogate optimal tree from a tree ensemble in Piccialli et al. (2024)



¹Bénard et al. (2021); Carrizosa et al. (2010, 2011, 2016, 2017, 2022a); Chevaleyre et al. (2013); Di Teodoro et al. (2024); Emine et al. (2024); Golea and Marchand (1993); Li et al. (2017); Martens et al. (2007); Piccialli et al. (2024); Vidal and Schiffer (2020)

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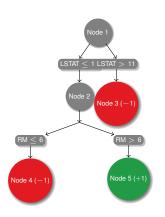
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Growing number of Explainable AI tools

Intelligent machines are asked to explain how their minds work

Researchers tackle problem that threatens to hold up adoption of advanced AI



A deep-learning machine on display at an artificial intelligence trade show in Tokyo last month © EPA

Richard Waters in San Francisco JULY 10 2017





Researchers at Parc, a laboratory with links to some of Silicon Valley's biggest breakthroughs, have just taken on a particularly thorny challenge: teaching intelligent machines to explain, in human terms, how their minds work.

The project, one of several sponsored by the US Defense Advanced Research Projects Agency (Darpa), is part of the search for an answer to one of the hardest problems in artificial intelligence.

Given an ML model \mathcal{M} , explain the prediction made for \boldsymbol{x}_0

- the important variables in the prediction of $\mathcal M$ for $x_0,$ or
- a surrogate and interpretable model, with high fidelity to M around x₀, or
- a counterfactual instance to x₀ but with a desired prediction by M

- Plethora of feature importance metrics, including SHapley Additive exPlanations (aka, SHAP) Lundberg and Lee (2017)
- Local Interpretable Model-Agnostic Explanations (aka, LIME)
 Ribeiro et al. (2016)
- Counterfactual Explanations
 Carrizosa et al. (2024); Martens and
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- Today, a collective framework for LIME, hereafter Collective Local Interpretable Model-Agnostic Explanations (CLIME)
- For a collection of instances, we build a surrogate model around each of them that is interpretable and locally accurate
- Our collective framework enables control over global properties of the explanations such as global feature selection, i.e., across the surrogate models





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- The prediction function $y : \mathbb{R}^p \to \mathcal{Y}$ associated with model \mathcal{M}
- The prediction function associated with surrogate model \hat{y} that explains y

Generalized Linear Models (GLMs) as interpretable surrogate

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- The simplest case is a linear model with vector of **coefficients** $\beta(\mathbf{x}) \in \mathbb{R}^p$

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so that $\hat{y}(\beta(\mathbf{x}), \tilde{\mathbf{x}}) \approx y(\tilde{\mathbf{x}})$. The fidelity of \hat{y} to y around \mathbf{x} can be measured, e.g., by

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A visualization of the output of CLIME

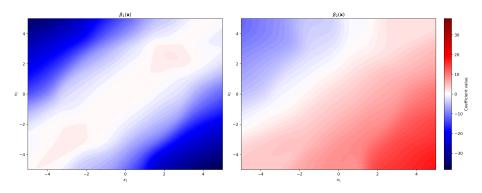


Figure: Visualizing $\beta_1(\mathbf{x})$ and $\beta_2(\mathbf{x})$ for a toy dataset, $y = (x_2 - x_1)^2 + 3x_1$.

The surrogate model (and the decision) in CLIME

- Although LIME mostly uses linear models independently of the nature of y, linear models are not suitable, for instance, in Supervised Classification
- GLMs are rich enough to deal with different types of response variable

$$g(\mathbb{E}[Y \mid \mathbf{X} = \tilde{\mathbf{x}}]) = \beta(\mathbf{x})^{\top} \tilde{\mathbf{x}},$$

where g is the so-called link function. Conversely,

$$\mathbb{E}[Y \mid \mathbf{X} = \tilde{\mathbf{x}}] = g^{-1}(\beta(\mathbf{x})^{\top}\tilde{\mathbf{x}})$$

• With this, by choosing the right g and fidelity, we can handle

Linear regfor continuous response for binary response *g* identity function **Logistic reg**for binary response *g* logistic function

Poisson reg for count data response g exponential function

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Linear reg	Logistic reg	Poisson reg
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As in LIME, we have a local measure of error:

$$\delta(\beta(\mathbf{x}),\mathbf{x}) := \int \omega(\mathbf{x},\tilde{\mathbf{x}}) \ell(y(\tilde{\mathbf{x}}),\hat{y}(\beta(\mathbf{x}),\tilde{\mathbf{x}})) \, d\mathbf{P}(\tilde{\mathbf{x}})$$

- Distribution P for ground truth of instances
 e.g., discrete uniform on a set of instances, or mixture of Normal distributions
- Weighting function $\omega : \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}_+$

$$e^{-\gamma \|\mathbf{x} - ilde{\mathbf{x}}\|_2^2} \qquad \qquad e^{-\gamma \|\mathbf{x} - ilde{\mathbf{x}}\|_2} \qquad \qquad \mathbb{1}_{\{\|\mathbf{x} - ilde{\mathbf{x}}\| \leq \epsilon\}}$$

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The objective function in CLIME measures the error incurred globally

$$\Delta(\beta) := \int \delta(\beta(\mathbf{x}), \mathbf{x}) d\mathbf{Q}(\mathbf{x}),$$

where **Q** is the distribution controlling the relevance of each explanation

The optimization model behind CLIME

The global optimization problem associated with CLIME reads as follows

$$\min_{\beta \in \mathcal{B}} \Delta(\beta),$$

where ${\cal B}$ is the feasible region for ${eta}$

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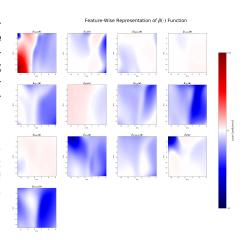
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Theoretical properties of CLIME

Our framework addresses some draw-backs of LIME pointed in the literature (Garreau and von Luxburg, 2020; Sepulveda et al., 2025; Tiukhova et al., 2024; Zhou and Wang, 2021). Indeed, if $\mathcal B$ consists of all functions, the following proposition can be shown:

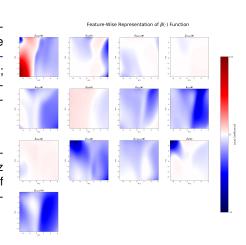
Proposition Under some technical conditions, if ω is continuous (resp., Lipschitz continuous) then the optimal solution of CLIME is continuous (resp., Lipschitz continuous)



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Cost-Sensitive Feature Selection

Our methodology is designed to be able to control global properties

Model \mathcal{M} may have used all p features, but, with a **stakeholder** point of view, we would like to know whether there are surrogate models using fewer features with a good error. For this, we now impose **global sparsity** to CLIME

This means that we would like to choose a subset of features $\mathcal{J} \subseteq \{1, \dots, p\}$, such that the surrogate models can only use features in \mathcal{J}

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The Cost-Sensitive Feature Selection Problem

The objective function of the Feature Selection Problem reads as follows $\pi(\mathcal{J}) := \min_{\beta_{\mathcal{J}}} \Delta_{\mathcal{J}}(\beta_{\mathcal{J}})$, with

- $\beta_{\mathcal{J}}$, $\mathbf{x}_{\mathcal{J}}$, $\tilde{\mathbf{x}}_{\mathcal{J}}$ be the respective projections of β , \mathbf{x} , $\tilde{\mathbf{x}}$
- $\bullet \ \delta_{\mathcal{J}}(\beta_{\mathcal{J}}(\mathbf{x}_{\mathcal{J}}), \mathbf{x}_{\mathcal{J}}) := \int \omega_{\mathcal{J}}(\mathbf{x}_{\mathcal{J}}, \tilde{\mathbf{x}}_{\mathcal{J}}) \ell(y(\tilde{\mathbf{x}}), \hat{y}_{\mathcal{J}}(\beta_{\mathcal{J}}(\mathbf{x}_{\mathcal{J}}), \tilde{\mathbf{x}}_{\mathcal{J}})) d\mathbf{P}(\tilde{\mathbf{x}})$
- $\bullet \ \Delta_{\mathcal{J}}(\beta_{\mathcal{J}}) := \int \delta(\beta_{\mathcal{J}}(\mathbf{x}_{\mathcal{J}}), \mathbf{x}_{\mathcal{J}}) \, d\mathbf{Q}(\mathbf{x}_{\mathcal{J}})$

Since the feature selection is cost-sensitive:

- Let c_j be the cost assigned to feature j (Carrizosa et al., 2008; Turney, 1995)
- Let C be the total budget

$$egin{array}{ll} egin{array}{ll} egin{array}{ll} egin{array}{ll} egin{array}{ll} egin{array}{ll} egin{array}{ll} \mathcal{J} \end{pmatrix} \ & ext{s.t.} & \sum_{j \in \mathcal{J}} c_j \leq C \end{array}$$

Assume that we have a solution \mathcal{J}^{cr} for the cost-sensitive feature selection problem. For instance, $\mathcal{J}^{cr} = \emptyset$, $\mathcal{J}^{cr} = \{1, \dots, p\}$ or \mathcal{J}^{cr} obtained with a greedy approach such as in stepwise feature selection procedures (Hastie et al., 2009)

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```
V_j^{\mathrm{cr}} \ := \ \begin{cases} \pi(\mathcal{J}^{\mathrm{cr}}) - \pi\big(\mathcal{J}^{\mathrm{cr}} \cup \{j\}\big), & j \notin \mathcal{J}^{\mathrm{cr}}, \\ \pi(\mathcal{J}^{\mathrm{cr}} \setminus \{j\}) - \pi(\mathcal{J}^{\mathrm{cr}}), & j \in \mathcal{J}^{\mathrm{cr}} \end{cases} \qquad \begin{array}{c} \texttt{11:} & \mathsf{Update feature set } \mathcal{J}^{\mathrm{cr}} \mathsf{ to its optimal solution} \\ \texttt{12:} & \mathsf{if } \pi(\mathcal{J}^{\mathrm{cr}}) < \pi(\mathcal{J}^{\mathrm{inc}}) \mathsf{ then} \\ \texttt{13:} & \mathcal{J}^{\mathrm{inc}} \leftarrow \mathcal{J}^{\mathrm{cr}} & \mathsf{b Update incumbent solution} \end{cases}
```

Assume that we have a solution \mathcal{J}^{cr} for the cost-sensitive feature selection problem. For instance, $\mathcal{J}^{cr}=\emptyset$, $\mathcal{J}^{cr}=\{1,\ldots,p\}$ or \mathcal{J}^{cr} obtained with a greedy approach such as in stepwise feature selection procedures (Hastie et al., 2009)

Let V_j^{cr} be a measure of the **contribution of feature** j **to** π around \mathcal{J}^{cr} . The linearization at $\mathcal{J} = \mathcal{J}^{cr}$ yields the following Knapsack Problem

$$egin{array}{ll} \max_{\mathcal{J}} & \sum_{j \in \mathcal{J}} V_j^{\mathsf{cr}} \ & \mathsf{s.t.} & \sum_{j \in \mathcal{J}} c_j \ \leq \ C \end{array}$$

```
How to measure V_j^{\text{cr}} g: \text{end for } 10^{\text{cr}} g: \text{end for } 10^{\text{cr}} f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the } 11^{\text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{color the Knapsack Problem with } V_j^{\text{cr}}, \text{ for all } j f: \text{colo
```

```
Algorithm Iterative Algorithm For Feature Selection

1: Input:
    Initial feature set \mathcal{J}^{\text{start}}
    Total budget C

2: Output: Selected feature set \mathcal{J}^{\text{inc}}
    3: Initialize:
4: Incumbent feature set \mathcal{J}^{\text{inc}} \leftarrow \mathcal{J}^{\text{start}}

5: Current feature set \mathcal{J}^{\text{orc}} \leftarrow \mathcal{J}^{\text{inc}}
    6: while stopping criteria not met do

7: for j=1 to p do

8: Calculate V_j^{\text{cr}}

9: end for

10: Solve the Knapsack Problem with V_j^{\text{orc}}, for all j

11: Update feature set \mathcal{J}^{\text{orc}} to its optimal solution

12: if \pi(\mathcal{J}^{\text{orc}}) < \pi(\mathcal{J}^{\text{inc}}) then
```

Assume that we have a solution \mathcal{J}^{cr} for the cost-sensitive feature selection problem. For instance, $\mathcal{J}^{cr} = \emptyset$, $\mathcal{J}^{cr} = \{1, \dots, p\}$ or \mathcal{J}^{cr} obtained with a greedy approach such as in stepwise feature selection procedures (Hastie et al., 2009)

Let $V_j^{\rm cr}$ be a measure of the **contribution of feature** j **to** π around $\mathcal{J}^{\rm cr}$. The linearization at $\mathcal{J}=\mathcal{J}^{\rm cr}$ yields the following Knapsack Problem

$$egin{array}{ll} \max_{\mathcal{J}} & \sum_{j \in \mathcal{J}} V_j^{\mathsf{cr}} \ & \mathsf{s.t.} & \sum_{j \in \mathcal{J}} c_j \ \leq \ C \end{array}$$

How to measure V_j^{cr} $V_j^{\text{cr}} := \begin{cases} \pi(\mathcal{J}^{\text{cr}}) - \pi(\mathcal{J}^{\text{cr}} \cup \{j\}), & j \notin \mathcal{J}^{\text{cr}}, \\ \pi(\mathcal{J}^{\text{cr}} \setminus \{j\}) - \pi(\mathcal{J}^{\text{cr}}), & j \in \mathcal{J}^{\text{cr}} \end{cases}$

Algorithm Iterative Algorithm For Feature Selection

```
1: Input:
    Initial feature set 𝑉start
    Total budget C
2: Output: Selected feature set 𝑉inc
3: Initialize:
4: Incumbent feature set 𝑉inc ← 𝑉start
5: Current feature set 𝑉inc ← 𝑉inc
6: while stopping criteria not met do
7: for / = 1 to p do
8: Calculate V<sup>cr</sup>
9: end for
10: Solve the Knapsack Problem with V<sup>cr</sup><sub>f</sub>, for all f
11: Update feature set 𝑉ar to its optimal solution
12: if π(𝑉ar) < π(𝑉inc) then
13: 𝑉inc ← 𝑉ar ▷ Update incumbent solution
14: end if
15: end while
```

Assume that we have a solution \mathcal{J}^{cr} for the cost-sensitive feature selection problem. For instance, $\mathcal{J}^{cr} = \emptyset$, $\mathcal{J}^{cr} = \{1, \dots, p\}$ or \mathcal{J}^{cr} obtained with a greedy approach such as in stepwise feature selection procedures (Hastie et al., 2009)

Let V_j^{cr} be a measure of the **contribution of feature** j **to** π around \mathcal{J}^{cr} . The linearization at $\mathcal{J} = \mathcal{J}^{\text{cr}}$ yields the following Knapsack Problem

$$egin{array}{ll} \max_{\mathcal{J}} & \sum_{j \in \mathcal{J}} V_j^{\mathsf{cr}} \ & \mathsf{s.t.} & \sum_{j \in \mathcal{J}} c_j \ \leq \ C \end{array}$$

How to measure V_j^{cr}

$$V_{j}^{\mathsf{cr}} \; := \; \begin{cases} \pi(\mathcal{J}^{\mathsf{cr}}) \; - \; \pi\big(\mathcal{J}^{\mathsf{cr}} \cup \{j\}\big), & j \notin \mathcal{J}^{\mathsf{cr}}, \\ \pi\big(\mathcal{J}^{\mathsf{cr}} \setminus \{j\}\big) - \pi\big(\mathcal{J}^{\mathsf{cr}}\big), & j \in \mathcal{J}^{\mathsf{cr}} \end{cases}$$

Algorithm Iterative Algorithm For Feature Selection

```
1: Input:
                Initial feature set .7 start
                 Total budget C
         2: Output: Selected feature set \mathcal{J}^{inc}
         3: Initialize:
         4: Incumbent feature set \mathcal{J}^{\text{inc}} \leftarrow \mathcal{J}^{\text{start}}

 Current feature set .7<sup>cr</sup> ← .7<sup>inc</sup>

          6: while stopping criteria not met do
                  for i = 1 to p do
                      Calculate V:
                  end for
          10: Solve the Knapsack Problem with V_i^{Cr}, for all j
         11: Update feature set \mathcal{J}^{cr} to its optimal solution
12: if \pi(\mathcal{J}^{cr}) < \pi(\mathcal{J}^{inc}) then 13: \mathcal{J}^{inc} \leftarrow \mathcal{J}^{cr}
                                                           end if
          15: end while
          16: return \mathcal{J}^{inc}
                                                              > Return the best set found
```

Datasets and CLIME ingredients

The ML model $\mathcal M$ to be explained is a Random Forest

Dataset	Response	# Observations	# Features	Surrogate Model
Boston Housing Regression	Continuous	506	13	Linear Reg
Communities and Crime	Continuous	810	103	Linear Reg
Law School	Continuous	20,800	17	Linear Reg
Boston Housing Classification	Binary	506	13	Logistic Reg
DebTrivedi	Count data	4,406	21	Poisson Reg

Table: Summary of benchmark datasets

Р	Q	ω	ℓ
Discrete uniform on dataset	Discrete uniform on dataset	$\omega(\mathbf{x}_0, \tilde{\mathbf{x}}) = e^{-\frac{1}{2}\ \mathbf{x}_0 - \tilde{\mathbf{x}}\ _2^2}$	see surrogate model

Table: CLIME ingredients

У	P	Q	ω	ŷ
RF	Discrete uniform on dataset	Discrete uniform on dataset	$\omega(\mathbf{x}_0, \tilde{\mathbf{x}}) = e^{-\frac{1}{2}\ \mathbf{x}_0 - \tilde{\mathbf{x}}\ _2^2}$	Linear Reg

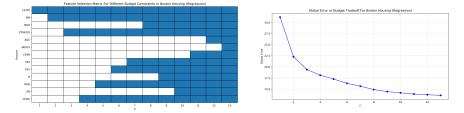
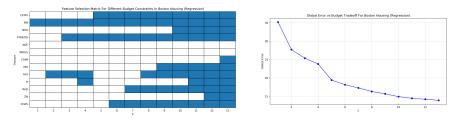


Figure: Boston Housing dataset (regression) with equal feature costs: Features chosen for all possible values of budget C (left), and global error vs budget (right)



 $\label{eq:Figure:Boston} \textit{Housing dataset (regression) with non-equal feature costs: Features chosen by Algorithm 1 for different values of budget <math>C$ (left), and global error vs budget (right)

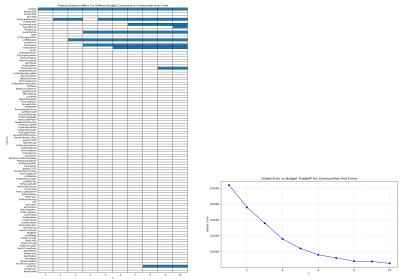


Figure: Communities and Crime dataset (regression) with equal feature costs: Features chosen by Algorithm 1 for budget $C \in \{1, 2, \dots, 10\}$ (left), and global error vs budget (right)

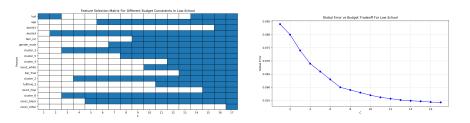


Figure: Law School (regression) dataset with equal feature costs: Features chosen by Algorithm 1 for all values of budget C (left), and global error vs budget (right)

CLIME: Classification

У	Р	Q	ω	ŷ
RF	Discrete uniform on dataset	Discrete uniform on dataset	$\omega(\mathbf{x}_0, \tilde{\mathbf{x}}) = e^{-\frac{1}{2}\ \mathbf{x}_0 - \tilde{\mathbf{x}}\ _2^2}$	Logistic Reg

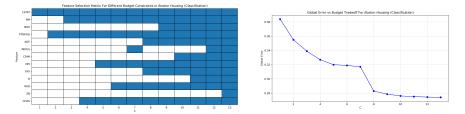


Figure: Boston Housing dataset (classification) with equal feature costs: Features chosen for all possible values of budget C (left), and global error vs budget (right)

CLIME: Poisson Regression

У	P	Q	ω	ŷ
RF	Discrete uniform on dataset	Discrete uniform on dataset	$\omega(\mathbf{x}_0, \tilde{\mathbf{x}}) = e^{-\frac{1}{2}\ \mathbf{x}_0 - \tilde{\mathbf{x}}\ _2^2}$	Poisson Reg

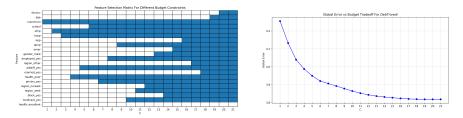


Figure: petrivedi (regression with count data) dataset with equal feature costs: Features chosen for all possible values of budget C (left), and global error vs budget (right)

Outline

Introduction

The CLIME methodology

- Global optimization formulation to enhance the explainability of ML models
- With a stakeholder point of view, we control global sparsity
- Very same methodology for different types of response variables through GLMs
- Currently, dealing with complex data (Blanquero et al., 2019, 2023) and other types of explainable surrogates (Carrizosa et al., 2021b)
- Explainability in other decision-making domains (De Bock et al., 2024), such as performance benchmarking (Benítez-Peña et al., 2020; Bogetoft et al., 2024)

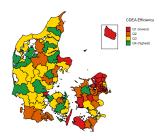
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References I

- S. Bach, G. Binder, A.and Montavon, F. Klauschen, K.-R. Müller, and W. Samek. On pixel-wise explanations for non-linear classifier decisions by layer-wise relevance propagation. PloS One, 10(7):e0130140, 2015.
- B. Baesens, R. Setiono, C. Mues, and J. Vanthienen. Using neural network rule extraction and decision tables for credit-risk evaluation. Management Science, 49(3):312–329, 2003.
- C. Bénard, G. Biau, S. Da Veiga, and E. Scornet. Sirus: Stable and interpretable rule set for classification. Electronic Journal of Statistics, 15(1):427–505, 2021.
- S. Benítez-Peña, P. Bogetoft, and D. Romero Morales. Feature selection in data envelopment analysis: A mathematical optimization approach. Omega, 96: 102068, 2020.
- D. Bertsimas, A. King, and R. Mazumder. Best subset selection via a modern optimization lens. The Annals of Statistics, 44(2):813-852, 2016.
- R. Blanquero, E. Carrizosa, A. Jiménez-Cordero, and B. Martín-Barragán. Variable selection in classification for multivariate functional data. *Information Sciences*, 481:445–462, 2019.
- R. Blanquero, E. Carrizosa, C. Molero-Río, and D. Romero Morales. On optimal regression trees to detect critical intervals for multivariate functional data. Computers and Operations Research, 152:106152, 2023.
- P. Bogetoft, J. Ramírez-Ayerbe, and D. Romero Morales. Counterfactual analysis and target setting in benchmarking. European Journal of Operational Research, 315(3):1083–1095, 2024.
- L. Breiman, Random forests, Machine Learning, 45(1):5-32, 2001.
- N. Burkart and M.F. Huber. A survey on the explainability of supervised machine learning. Journal of Artificial Intelligence Research, 70:245–317, 2021.
- E. Carrizosa, B. Martín-Barragán, and D. Romero Morales. Multi-group support vector machines with measurement costs: A biobjective approach. Discrete Applied Mathematics, 156:950–966, 2008.
- E. Carrizosa, B. Martín-Barragán, and D. Romero Morales. Binarized support vector machines. INFORMS Journal on Computing, 22(1):154–167, 2010.
- E. Carrizosa, B. Martin-Barragán, and D. Romero Morales. Detecting relevant variables and interactions in supervised classification. European Journal of Operational Research, 213(1):260–269, 2011.
- E. Carrizosa, A. Nogales-Gómez, and D. Romero Morales. Strongly agree or strongly disagree?: Rating features in Support Vector Machines. Information Sciences, 329:256–273, 2016.
- E. Carrizosa, A. Nogales-Gómez, and D. Romero Morales. Clustering categories in support vector machines. *Omega*, 66:28–37, 2017.
- E. Carrizosa, M. Galvis Restrepo, and D. Romero Morales. On clustering categories of categorical predictors in generalized linear models. Expert Systems With Applications, 182:115245, 2021a.
- E. Carrizosa, C. Molero-Río, and D. Romero Morales. Mathematical optimization in classification and regression trees. TOP, 29(1):5–33, 2021b.
- E. Carrizosa, K. Kurishchenko, A. Marín, and D. Romero Morales. Interpreting clusters by prototype optimization. Omega, 107:102543, 2022a.

References II

- E. Carrizosa, L.H. Mortensen, D. Romero Morales, and M.R. Sillero-Denamiel. The tree based linear regression model for hierarchical categorical variables. Expert Systems With Applications, 203(7):117423, 2022b.
- E. Carrizosa, K. Kurishchenko, A. Marin, and D. Romero Morales. On clustering and interpreting with rules by means of mathematical optimization. Computers & Operations Research, 154:106180, 2023.
- E. Carrizosa, J. Ramírez-Ayerbe, and D. Romero Morales. Mathematical optimization modelling for group counterfactual explanations. European Journal of Operational Research, 319(2):399–412, 2024.
- E. Carrizosa, T. Halskov, and D. Romero Morales. Collective LIME: Making black boxes explainable and sparse. Technical report, Copenhagen Business School, Denmark, https://www.researchgate.net/publication/394413617 Collective LIME Making black boxes explainable and sparse,
- ntcps://www.researchgate.net/publication/39441361/_collective_htmp_making_black_boxes_explainable_and_sparse, 2025a.

 E. Carrizosa, K. Kurishchenko, and D. Romero Morales. On enhancing the explainability and fairness of tree ensembles. European Journal of Operational
- Research, 323(2):599–608, 2025b.

 T. Chen and C. Guestrin. XGBoost: A scalable tree boosting system. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge
- Discovery and Data Mining, pages 785–794, 2016.
- Y. Chevaleyre, F. Koriche, and J.-D. Zucker. Rounding methods for discrete linear classification. In S. Dasgupta and D. McAllester, editors, Proceedings of the 30th International Conference on Machine Learning (ICML-13), volume 28, pages 651–659. JMLR Workshop and Conference Proceedings, 2013.
- K.W. De Book, K. Coussement, A. De Caigny, R. Slowiński, B. Baesens, R.N. Boute, T.-M. Choi, D. Delen, M. Kraus, S. Lessmann, S. Maldonado, D. Martens, M. Obkarsdóttir, C. Vairetti, W. Verbeke, and R. Weber. Explainable Al for Operational Research: A defining framework, methods, applications, and a research agenda. European Journal of Operational Research, 317(2):249–272, 2024.
- G. Di Teodoro, M. Monaci, and L. Palagi. Unboxing tree ensembles for interpretability: a hierarchical visualization tool and a multivariate optimal re-built tree. EURO Journal on Computational Optimization, 12:100084, 2024.
- Y. Emine, A. Forel, I. Malek, and T. Vidal. Free lunch in the forest: Functionally-identical pruning of boosted tree ensembles. arXiv preprint arXiv:2408.16167, 2024.
- D. Garreau and U. von Luxburg. Explaining the Explainer: A First Theoretical Analysis of LIME. In S. Chiappa and R. Calandra, editors, Proceedings of the Twenty Third International Conference on Artificial Intelligence and Statistics, volume 108 of Proceedings of Machine Learning Research, pages 1287–1296, 2020.
- A. Goldstein, A. Kapelner, J. Bleich, and E. Pitkin. Peeking inside the black box: Visualizing statistical learning with plots of individual conditional expectation. Journal of Computational and Graphical Statistics, 24(1):44–65, 2015.
- M. Golea and M. Marchand. On learning perceptrons with binary weights. Neural Computation, 5(5):767-782, 1993.
- I. Goodfellow, Y. Bengio, and A. Courville. Deep Learning. MIT Press, 2016.
- T. Hastie, R. Tibshirani, and J. Friedman. The Elements of Statistical Learning. Springer, New York, 2nd edition, 2009.

References III

- H. Hazimeh and R. Mazumder. Fast best subset selection: Coordinate descent and local combinatorial optimization algorithms. *Operations Research*, 68(5): 1517–1537, 2020.
- O. Li, H. Liu, C. Chen, and C. Rudin. Deep learning for case-based reasoning through prototypes: A neural network that explains its predictions, 2017.
- S.M. Lundberg and S.-I. Lee. A unified approach to interpreting model predictions. In Advances in Neural Information Processing Systems, pages 4765–4774, 2017.
- D. Martens and F. Provost. Explaining data-driven document classifications. MIS Quarterly, 38(1):73-99, 2014.

interpretability. Expert Systems with Applications, 274:126922, 2025.

- D. Martens, B. Baesens, T.V. Gestel, and J. Vanthienen. Comprehensible credit scoring models using rule extraction from support vector machines. European Journal of Operational Research, 183(3):1466–1476, 2007.
- J.A. Nelder and R.W.M. Wedderburn. Generalized linear models. Journal of the Royal Statistical Society Series A: Statistics in Society, 135(3):370–384, 1972.
- C. Panigutti, R. Hamon, I. Hupont, D. Fernandez Llorca, D. Fano Yela, H. Junklewitz, S. Scalzo, G. Mazzini, I. Sanchez, J. Soler Garrido, and E. Gomez. The Role of Explainable AI in the Context of the AI Act. In Proceedings of the 2023 ACM Conference on Fairness, Accountability, and Transparency, FAccT '23, pages 1139–1150, New York, NY, USA, 2023.
- V. Piccialli, D. Romero Morales, and C. Salvatore. Supervised feature compression based on counterfactual analysis. European Journal of Operational Research, 317:273–285, 2024.
- M.T. Ribeiro, S. Singh, and C. Guestrin. "Why Should I Trust You?": Explaining the Predictions of Any Classifier. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, pages 1135–1144, 2016.
- C. Rudin, C. Chen, Z. Chen, H. Huang, L. Semenova, and C. Zhong. Interpretable machine learning: Fundamental principles and 10 grand challenges. Statistics Surveys, 16:1–85, 2022.
- Statistics Surveys, 16:1–85, 2022.

 E. Sepulveda, F. Vandervorst, B. Baesens, and T. Verdonck. Enhancing explainability in real-world scenarios: Towards a robust stability measure for local
- R. Tibshirani. Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society: Series B (Methodological), 58(1):267-288, 1996.
- E. Tiukhova, P. Vemuri, N. López Flores, A.S. Islind, M. Óskarsdóttir, S. Poelmans, B. Baesens, and M. Snoeck. Explainable learning analytics: Assessing
- the stability of student success prediction models by means of explainable ai. *Decision Support Systems*, 182:114229, 2024.

 P.D. Turney. Cost-sensitive classification: Empirical evaluation of a hybrid genetic decision tree induction algorithm. *Journal of Artificial Intelligence Research*, 2:369–409, 1995.
- B. Ustun and C. Rudin. Supersparse linear integer models for optimized medical scoring systems. Machine Learning, 102(3):349-391, 2016.
- b. Ostun and C. Rudin. Supersparse linear integer models for optimized medical scoring systems. *Machine Learning*, 102(3):349–391, 2016
- T. Vidal and M. Schiffer. Born-again tree ensembles. In International Conference on Machine Learning, pages 9743–9753. PMLR, 2020.
- Wachter, B. Mittelstadt, and C. Russell. Counterfactual explanations without opening the black box: Automated decisions and the GDPR. Harvard Journal of Law & Technology, 31:841–887, 2017.
- G. Zhou, Z.and Hooker and F. Wang. S-LIME: Stabilized-LIME for model explanation. In Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining, pages 2429–2438, 2021.